Optimal incentives for heterogeneous spillover effects*

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Abstract

We develop a principal-agent model with heterogeneous spillover effects. The optimal incentive scheme rewards collective signals for agents with positive spillovers, while rewarding relative signals for these with negative spillovers. We characterize the optimal effort pair and identify sufficient conditions under which full efforts (a partial effort) are optimal, which depend on the intensity and direction of spillovers, signal noise, and revenue cross-difference. These results offer guidance for incentive design in organizations with interdependent performance metrics and heterogeneous cross-agent influences.

Keywords: Principal–agent model, Heterogeneous spillover effects, Contract design.

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1 Introduction

In many modern organizations, agents' actions are technologically or operationally intertwined, creating spillover effects—where one agent's effort also stochastically influences another's output, so a single agent's output can serve as a noisy signal of the efforts of multiple agents. These effects appear across a wide range of settings: in software engineering, effort invested by engineers for product refinement can impact salesmen's efficiency in promoting the software (Song et al., 1996; Kekre et al., 1995); in healthcare, a nurse's attentiveness may speed up or slow down patients' recovery and hence impact a physician's efficiency (Aiken et al., 2002); in customer support, one agents' reply speed and quality can ease or burden coworkers' workloads. Even in knowledge industries like education, consulting, or software, peer effort regularly alters colleagues' output through shared resources, workflow dependencies, or mentoring (Ibrahim et al., 2016). Crucially, these effects are often heterogeneous: the strength, direction, and nature of the impact from one agent to another may differ in the two directions.

Among these heterogeneities, one extreme case is that two agents have spillover effects in opposite directions, which we refer to as asymmetric spillover effects. A stylized example illustrates both this asymmetry and the importance of effort-pair selection. Consider a digital firm in which the technology department is evaluated by the number of innovations, such as new features deployed, while the compliance department is evaluated by the number of violations detected. When the technology department works intensively, it not only increases the probability of releasing more features, but also gives compliance more material to scrutinize, thereby increasing the probability that violations are detected—a positive spillover from technology to compliance. Hence, violations detected may also be informative about the technology department's effort. Conversely, when compliance imposes more procedural checks and tightens review standards, it increases the probability of detecting violations but may also delay or block feature deployment — a negative spillover from compliance to technology. Hence, fewer features deployed may also be informative about high compliance effort. In summary, features deployed and violations detected are noisy signals of both departments' efforts, but in different directions.[1]

Deviating from the standard homogeneous cases, this example with asymmetric spillover effects raises concerns about the optimal effort pair, in addition to the contract shape. In the homogeneous setting, in a binary-effort model where agents can only take high or low effort, necessarily the same effort is induced from both agents, and only the case where high effort is induced from both is of interest. However, the optimal effort pair becomes less trivial when agents are heterogeneous. In the

[1] Although the classical principal-agent model describes the relationship between a principal and individual agents, one can interpret agents here as department heads who receive team-level compensation and redistribute it internally. This formulation naturally generalizes the classic model to the organizational context.

technology-compliance example, suppose that high effort by the technology department is indispensable. A partial effort (only one agent exerting a high effort) is desirable when a firm aims to maximize its probability of an innovative breakthrough, and where intensive compliance effort adds little to the effective value of technological innovation. Conversely, full efforts are desirable when high compliance effort increases the effective value of technological innovation: rapid innovation may generate many new features, but these features are valuable to the firm only if they can be safely launched and sustained in the market. However, despite empirical studies in organizational economics that have documented such productive asymmetries (Ichniowski and Shaw, 2003; Insead, 2019), most theoretical work continues to assume high efforts as optimal with symmetric or homogeneous interactions. This motivates our central question: when should a principal induce full efforts (a partial effort) with (heterogeneous) spillover effects?

Motivated by the above, the contribution of this paper is two-fold. On the one hand, we aim to identify the optimal incentive scheme for heterogeneous spillover effects and its comparative statics with respect to the intensity of spillover effects. We investigate the principal–agent relationship in a setting where one agent's effort can positively or negatively affect the other agent's observed output stochastically, and where this spillover effect need not be symmetric in either intensity or direction. In particular, we extend the benchmark model of Fleckinger et al. (2024) to a principal–agent model with two agents who have heterogeneous spillover effects on each other, where these spillover effects may moreover be asymmetric.[2]

On the other hand, we endogenize the principal's choice of effort levels and analyze when it is optimal to induce full efforts versus a partial effort. Rather than assuming that high effort from both agents is always desirable, we ask when the principal should induce full efforts over a partial effort. This deviates from the focus of the benchmark model of Fleckinger et al. (2024), where the principal's desired effort profile is fixed at full effort, and the main problem is to characterize the profit-maximized incentive scheme. In our setting, the principal compares the added revenue from inducing an additional high effort with the extra incentive cost required to induce it. Under this setting, we derive simple conditions for spillover intensity, signal noise, and revenue complementarity/substitutability that determine the optimal effort pair.

Our first set of results characterizes the cost-minimizing incentive scheme for a given effort profile. Under limited liability and risk neutrality, the principal optimally conditions each agent's pay on both the agent's own signal and the other agent's signal whenever a spillover exists between their tasks. In particular, a positive bonus is placed only on the most cost-efficient signal pair to induce high effort in the optimal contract, which may vary across agents. For instance, going back to the ex-

[2]We include in the supplementary material that it is feasible to extend our argument to $n \geq 3$, but we limit our discussion to $n = 2$ for simplicity.

ample of the technological and compliance departments: a positive bonus would be assigned for the technological department if both departments receive a high signal, whereas a positive bonus would be assigned for the compliance department if it receives a high signal and the technological department receives a low signal instead. This is because both a high number of deployed new features and a high number of violations make it more likely that the technological department made a high effort; the technological department is rewarded when both departments perform well. As a high number of violations and a lower number of deployed features both make it more likely that the compliance department made a greater effort, the compliance department is rewarded when it performs well, and the technological department does not.

Our second set of results concerns the principal's optimal effort pair. We compare the expected profit from inducing full efforts with that from inducing a partial effort by specifying the added revenue and extra cost. We show that if each agent's effort increases the marginal benefit of the other's effort in relative terms sufficiently (i.e., agents' efforts are not log probabilistically substitutes, and are not overall substitutes), inducing full efforts is optimal. Conversely, if each agent's effort decreases the marginal benefit of the other's effort in relative terms (i.e., agents' efforts are not log probabilistically complements, and are not overall complements), a partial effort is optimal. Comparative statics concerning both cost and revenue show that full efforts become relatively more profitable if spillover effects are more intense when the other agent invests high effort, and if the signals received by the principal about effort are more informative under high effort (low noise). In contrast, if the spillover effects are weak or if higher effort greatly increases the informational cost (because signals received by the principal remain noisy), the extra effort is less valuable and hence the principal is more inclined to induce a partial effort.

The remainder of the paper is structured as follows. Section 2 delivers connections to related literature. Section 3 introduces the model setting and derives the optimal payments under any combination of spillovers, including heterogeneous ones. Section 4 analyzes the principal's choice of optimal effort pairs through a comparative-statics exercise and reaches sufficient conditions for optimality of full efforts. Section 5 concludes the paper, summarizing the findings and their implications.

2 Literature Review

This paper is related to the broad literature on multi-agent incentives with interdependent outcomes. A central insight in this literature is that an agent's own output is not always the only relevant signal for incentive provision. When outputs are correlated, or when one agent's effort affects another agent's output, the principal may optimally condition compensation on several agents' performance measures. The relevant question is then, what are the corresponding optimal incentive schemes, and the optimal effort pair with the presence of these spillover effects? Our paper builds on this insight, but focuses on technological spillover effects: one agent's effort changes the distribution of the other agent's signal, rather than the related signals. Within this framework, we study both the optimal form of compensation and the optimal effort profile to be induced.

One strand of the literature studies multi-agent incentives with interdependent outcomes mainly through informational spillover effects or correlated signals. In the classical team moral-hazard setting, Holmström (1982) shows that individual performance evaluation may be suboptimal when other agents' outputs contain information relevant for incentive provision. Ramakrishnan and Thakor (1991) compare Independent Performance Evaluation (IPE hereafter), Relative Performance Evaluation (RPE hereafter), and Collective Performance Evaluation (CPE hereafter) in a two-agent agency model with conditionally correlated outputs. They show that CPE may dominate when informational spillover effects are sufficiently low, whereas RPE may dominate when they are sufficiently high. Che and Yoo (2001) also studies optimal incentives in teams and shows that low-powered group incentives can be optimal when team interaction and implicit incentives help sustain cooperation. Our paper differs from this strand by abstracting from informational spillover effects and instead focusing on technological ones, in which one agent's effort directly changes the probability distribution of another agent's signal.

Another closely related strand studies how technological spillover effects impact the efficiency of choosing CPE and RPE. Itoh (1991) analyzes incentives in multi-agent settings and shows that CPE can be optimal when agents' efforts have positive spillover effects through mutual help. Drago and Turnbull (1988) similarly shows that tournament-style RPE is weakened when workers exert positive externalities on one another. Choi (1993) takes a step forward by considering both informational and technological spillovers and shows that the optimality of CPE/RPE depends on the relative strength of technological and informational externalities. These papers establish that the sign and strength of spillover effects matter for the optimal contract form. Our analysis follows this line by linking positive spillovers to CPE and negative spillovers to RPE, but extends it by allowing spillovers to be heterogeneous across agents and by endogenizing the principal's desired effort profile.

The closest papers to ours are Fleckinger (2012) and Fleckinger et al. (2024). Assume risk-averse agents and priority of inducing full efforts, Fleckinger (2012) studies the optimality of CPE and RPE in a moral-hazard model with both technological and informational spillover effects. Instead of assuming risk-averse agents, Fleckinger et al. (2024) further extends the discussion to potential collusion behavior and the accuracy effect for measurement by posing a limited liability assumption. We build on this framework but change the object of analysis. First, we allow technological spillovers to differ in direction and intensity across agents, so that one agent may call for collective performance evaluation while the other calls for relative performance evaluation. Second, we do not take full efforts as given. Instead, we ask when full effort is optimal relative to partial effort, and how this choice depends on spillover intensity, signal noise, and the revenue cross-difference.

Our paper is also related to work that studies interdependent incentives through cooperation, relational considerations, behavioral mechanisms, or network interactions. Livio and De Chiara (2019) study a behavioral-contracting environment in which interdependent incentives can trigger reciprocal behavior among agents. Dasaratha et al. (2024) study incentive design with spillovers in a network setting and show that optimal incentive pay depends not only on agents' productivity but also on their organizational centrality and responsiveness to incentives. These papers highlight mechanisms through which agents' incentives become linked across an organization. Our model abstracts from social preferences, repeated-game considerations, and general network structures, and instead isolates a technological channel in a two-agent environment: the effort of one agent changes the informativeness and value of the other agent's performance signal.

3 Principal-agent model with heterogeneous spillover effects

In general, the section outlines a procedure for contract design if heterogeneous spillover effects are present and high effort from both agents is always desirable.[3] The principal wants agents to act accordingly by designing specific incentive schemes, while agents determine their effort levels to maximize personal payoffs. Proposition 1 formalizes the resulting optimal incentive scheme, which is particularly relevant for start-up firms. Consider, for example, a start-up pursuing a *burning money* strategy to build market awareness, expand market share, eliminate competitors, and attract further investment. Such firms often promote products despite negative marginal revenue, invest heavily in advertising, and boost social media visibility—all of which require sustained high effort from multiple departments (Berlin, 2025). In this setting, the principal designs incentive schemes that incentivize high effort at a minimal cost to the principal, while agents allocate effort to optimize their own payoffs.

3.1 Model

There are two heterogeneous agents, 1 and 2, where we use symbols $i = 1, 2$ and $j \neq i$, to refer to the agents. For simplicity, efforts (e_i and e_j) are binary, and each agent simultaneously chooses whether or not to exert effort, $e_i \in \{0, 1\}$, at a cost $c \cdot e_i$ where $c > 0$. The extension to n agents is shown in the Section 2 of the Supplementary Material. Following the same approach as Fleckinger et al. (2024), agents and the principal are assumed to be risk-neutral for simplicity. Efforts invested by

[3] For simplicity, we refer to a pair of spillover effects as asymmetric if they go in opposite directions.

agents are not observed by the principal or the other agent unless specified otherwise. This indicates that the principal has to rely on signals to infer which effort level agents invested, and agents make their decisions simultaneously. The principal observes a signal ($R_i \in \{H, L\}$) about agent i's effort. However, the signal received about the effort of agent j is also informative about the effort of agent i. For this reason, it makes sense to say that $\mathbf{R}_i = (R_i, R_j) \in [L, H] \times [L, H]$ is the signal pair one receives about the effort of agent i.[4] The signals are assumed to be verifiable and serve as the sole variables available for contracting. Denote by $Pr_i(\mathbf{R}_i|e_i, e_j)$ the probability of observing signals $\mathbf{R}_i$ when the agent i invests effort level e_i and agent j invests effort level e_j.[5]

To elicit efforts from agents, the principal has to design an incentive scheme. Since each signal might reveal information for both agents, the compensation scheme can be made contingent on the whole signal pair $\mathbf{R}_i$. An incentive scheme for agent i can be formalized as follows:

$$\omega_i = \{\boldsymbol{\omega_i}(\mathbf{R}_i)\} \text{ for } \forall \mathbf{R}_i \in [L, H] \times [L, H]$$

where $\omega_i(\cdot)$ is a scalar function that projects signal statuses to a real number. More specifically, wage profiles takes the form $\omega_i(\cdot) : \mathfrak{R}_i^2 \to \mathbb{R}^+$, where $\mathfrak{R}_i^2 = [L, H] \times [L, H]$ and $\mathbb{R}^+$ represents the set of non-negative real numbers. Given an incentive scheme $\boldsymbol{\omega_i}$ and a pair of efforts (e_i, e_j), agent i's expected payoff can be defined as

$$U_i(\boldsymbol{\omega_i}|e_i, e_j) = E_{\mathfrak{R}_i^2}[\boldsymbol{\omega_i}(\mathbf{R}_i)|e_i, e_j] - c \cdot e_i$$

where $U_i(\cdot)$ represents the utility function for agent i as the expected payment minus the cost of investing effort e_i.

3.2 Principal's problem

Under the conventional principal-agent framework, the primary purpose of the principal is to optimize expected profits by maximizing the expected revenue earned, less the expected payment to agents, through structuring appropriate incentive schemes that motivate agents to exert effort. To keep our results general and simplify the case, this model starts by assuming that the principal always wants both agents to exert high effort ($(e_i^*, e_j^*) = (1, 1)$), following the same reasoning as Fleckinger et al. (2024), and further analysis for other cases will follow in the subsequent subsections. The principal's expected revenue of inducing the effort pair (1, 1) minus the expected payment is thus assumed to

[4] Note that in this setting, if H is received for agent i and L for agent j, this same event is denoted as a signal pair $\mathbf{R}_i = (H, L)$ on the effort of player i, and as a signal pair $\mathbf{R}_j = (L, H)$ on the effort of player j.

[5] This means that $Pr_i(L, H|e_i, e_j) = Pr_j(H, L|e_j, e_i)$ and these probabilities are assumed non-zero for simplicity unless otherwise specified.

be greater than the principal's expected profits with any other effort configuration.[6] Therefore, the principal's problem has now shifted from maximizing expected profits to minimizing the expected payment of inducing the effort pair (1, 1).

Besides this assumption, the principal must meet several other constraints while structuring incentive schemes to ensure agents are willing to participate and act accordingly. The first is the participation constraint. This means agents would only accept wages at least as high as their reservation utilities. For simplicity, the reservation utilities are assumed to be zero following the same assumption by Fleckinger et al. (2024). Therefore, the participation constraints can be written as follows:

$$E_{\mathfrak{R}_i^2}[\omega_i(\mathbf{R}_i)|1,1] - c \geq 0 \text{ for } i = 1,2 \tag{1}$$

Another constraint is limited liability, meaning payments to agents cannot be negative. This reflects the limited wealth of agents, preventing them from making payments to the principal, as well as some institutional settings that do not permit negative wages. Therefore, this article introduces the limited liability constraints:

$$\omega_i(\mathbf{R}_i) \geq 0 \text{ for } \forall i = 1,2 \text{ with } \forall \mathbf{R}_i \in [L,H] \times [L,H] \tag{2}$$

The last constraint for extracting high effort levels is the incentive constraint, saying that the principal must set the expected utility to the agent of investing in high effort no lower than the expected utility of investing in low effort, which means that, conditional on agent j doing high effort, agent i should pick high effort rather than low effort:

$$E_{\mathfrak{R}_i^2}[\omega_i(\mathbf{R}_i)|1,1] - c \geq E_{\mathfrak{R}_i^2}[\omega_i(\mathbf{R}_i)|0,1] \text{ for } i = 1,2 \tag{3}$$

Thus, the payment minimization problem can be expressed as

$$\min_{\omega_i(\mathbf{R}_i),\omega_j(\mathbf{R}_j)} E_{\mathfrak{R}_i^2}[\omega_i(\mathbf{R}_i)|1,1] + E_{\mathfrak{R}_i^2}[\omega_j(\mathbf{R}_j)|1,1] \text{ , subject to (1), (2) and (3)} \tag{4}$$

which aims at finding the most cost-efficient wage profile for each agent to extract high effort.

3.3 Optimal incentive schemes for heterogeneous spillover effects

To find the solution to the principal's problem, first define the incentive efficiency for a certain pair of signals as a tool to derive the optimal incentive schemes.

[6] One can also consider it as as a first step towards knowing what effort pair is optimal for the principal to induce, where one first needs to know what payments the principal would set for each effort pair. So, to determine whether inducing effort (1, 1) is optimal, one must first understand the payment the principal would make if inducing (1, 1).

Definition 1. *For agent i at signal pair $\boldsymbol{R}_i$, incentive efficiency $I_i(\boldsymbol{R}_i)$ is defined as*

$$I_i(\boldsymbol{R}_i) \equiv \frac{Pr_i(\boldsymbol{R}_i|1,1) - Pr_i(\boldsymbol{R}_i|0,1)}{Pr_i(\boldsymbol{R}_i|1,1)} \text{ for } i = 1,2$$

Therefore, the incentive efficiency is a measure of how an agent's marginal incentives to exert effort are related to the principal's marginal cost of inducing such effort (Fleckinger et al., 2024). Under this setting, one can obtain Lemma 1. One can also learn the true meaning behind the incentive efficiency. The denominator is the coefficient of the payment for R_i in the principal's costs, while the numerator arises in the Lagrangian through the incentive constraint.

Lemma 1. *Under risk neutrality and limited liability, an optimal incentive scheme that solves Problem (4) entails positive wages only for signal pair(s) $\boldsymbol{R}_i^*$ with the highest incentive efficiency $I_i(\boldsymbol{R}_i^*)$ for each agent $i = 1, 2$.*

Please note that all proofs are in the Section 3 of the Supplementary Material. As is typical in models with limited liability constraints and risk neutrality, Lemma 1 shows that the optimal incentive scheme is a single corner solution because the maximand and constraints are all linear. Lemma 1 not only suggests a strong link between incentive efficiency and positive wage profiles, but it also proves that the optimal wage profiles can differ across agents by introducing heterogeneity. Specifically, it is possible that, $R_i^{i,*}$ is different from $R_i^{j,*}$, and/or that $R_j^{i,*}$ is different from $R_j^{j,*}$, where $\mathbf{R}_i^* = (R_i^{i,*}, R_j^{i,*})$ and $\mathbf{R}_j^* = (R_j^{j,*}, R_i^{j,*})$ represent signal pairs for each agent with the highest incentive efficiency, $I_i(\mathbf{R}_i^*)$.

Following Lemma 1, the next step of characterizing the optimal incentive scheme is to compare the incentive efficiency across signal pairs. Two key assumptions will be employed throughout the analysis:

Assumption 1. *For $\boldsymbol{R}_i \in \Re_i^2$, the joint probability function can be disentangled into the product of two separate probabilities, referred to as informational independence[7]:*

$$Pr_i(\boldsymbol{R}_i|e_i, e_j) = Pr_i(R_i|e_i, e_j)Pr_j(R_j|e_j, e_i) \tag{5}$$

where $\boldsymbol{R}_i = (R_i, R_j)$ represents the signal pair and $Pr_i(R_i|e_i, e_j)$ represents the probability of reaching signal R_i for agent i given effort pair (e_i, e_j), with $Pr_j(R_j|e_j, e_i)$ defined analogously.

[7]Please note that it is also referred to as *stochastic independence* in Itoh (1991). We refer to it as informational independence throughout the remainder of this article for consistency.

Assumption 2. *For* $\forall i \in \{1, 2\}$*, the probability of receiving an* H *signal follows the inequality:*

$$Pr_i(H|1, 1) > Pr_i(H|0, 1) \tag{6}$$

which is equivalent to the monotone likelihood ratio property (MLRP) under Assumption 1.

Please note that Assumption 2 is equivalent to $\frac{Pr_i(H,S|1,1)}{Pr_i(H,S|0,1)} > \frac{Pr_i(L,S|1,1)}{Pr_i(L,S|0,1)}$ if information independence is assumed. With Assumptions (1) and (2), Lemma 2 follows from Lemma 1 and further pinpoints the shape of the optimal incentive scheme.

Lemma 2. *Under informational independence (Assumption 1), and monotone likelihood ratio property (Assumption 2), the principal never sets a positive wage for agent* i *if* $R_i = L$*, where* $i = 1, 2$*.*

This helps rule out the possibility that signal pairs starting with L have the highest incentive efficiency. To further disentangle which signal pair(s) bear the most incentive efficiency, we define the spillover effect.

Definition 2. *A positive spillover effect of agent* i *on agent* j *means*

$$Pr_j(H|1, 1) > Pr_j(H|1, 0) \tag{7}$$

A negative spillover effect of agent i *on agent* j *means*

$$Pr_j(H|1, 1) < Pr_j(H|1, 0) \tag{8}$$

No spillover effect of agent i *on agent* j *means*

$$Pr_j(H|1, 1) = Pr_j(H|1, 0) \tag{9}$$

One example of a positive spillover effect within firms is that a high effort from the technological department would make promoting the product easier for the marketing department, as indicated by Bharadwaj et al. (2007), and therefore increase the likelihood of achieving a high signal for the marketing department. Regarding a negative spillover effect, consider the relationship between the technological and compliance departments. The compliance department ensures that the development procedure adheres closely to the regulations. High effort invested by the compliance group would lower the probability of reaching a high signal for the technological department, as they impose more constraints (Goldfarb and Tucker, 2011).

Define three different types of contracts based on the inequalities between wage profiles, following the same idea as in Fleckinger et al. (2024). However, what differs in our setting is the heterogeneity among agents. As we allow for heterogeneity among agents, the optimal incentive scheme for two agents may be different.

Definition 3. *Relative Performance Evaluation (RPE) arises for agent i if:*

$$\omega_i(H,H) < \omega_i(H,L)$$

Accordingly, Collective Performance Evaluation (CPE) arises for agent i if:

$$\omega_i(H,H) > \omega_i(H,L)$$

Accordingly, Independent Performance Evaluation (IPE) arises for agent i if:

$$\omega_i(H,H) = \omega_i(H,L)$$

Equipped with informational independence (1) and MLRP (2) , one can derive Proposition 1 regarding the optimal incentive scheme for different types of contracts.

Proposition 1. *Under informational independence (Assumption 1), and monotone likelihood ratio property (Assumption 2), an optimal incentive scheme entails*

(i) Collective Performance Evaluation ($\omega_i^*(H,H) > \omega_i^*(H,L) = 0$), if agent i has a positive spillover effect on agent j ($Pr_j(H|1,1) > Pr_j(H|1,0)$).

(ii) Relative Performance Evaluation ($\omega_i^*(H,L) > \omega_i^*(H,H) = 0$), if agent i has a negative spillover effect on agent j ($Pr_j(H|1,0) > Pr_j(H|1,1)$).

(iii) Independent Performance Evaluation ($\omega_i^*(H,L) = \omega_i^*(H,H) > 0$), if agent i has no spillover effect on agent j ($Pr_j(H|1,0) = Pr_j(H|1,1)$).

Specifically, the optimal incentive scheme takes the form:

$$\begin{cases} \omega_i^*(\boldsymbol{R}_i^*) = \frac{c}{Pr_i(\boldsymbol{R}_i^*|1,1) - Pr_i(\boldsymbol{R}_i^*|0,1)} \\ \omega_i^*(\boldsymbol{R}_i^{-*}) = 0 \end{cases}$$

with the expected payment equal to $\frac{c}{I_i(\boldsymbol{R}_i^*)}$.

$\boldsymbol{R}_i^*$ represents signal pair(s) attached with the highest incentive efficiency $I_i(\boldsymbol{R}_i^*)$, which is determined by the inequality in (i), (ii), and (iii), and $\boldsymbol{R}_i^{-*}$ represents the signal pair other than $\boldsymbol{R}_i^*$.

One can interpret these results from two perspectives. As the optimal incentive scheme is always tied to incentive efficiency, Assumption 2 and the direction of the spillover effect determine which signal pair has the highest incentive efficiency, and will receive payment. Among them, MLRP ($Pr_i(H|1,1) > Pr_i(H|0,1)$) determines that the agent should never be paid for his own L signal as stated in Lemma 2. Spillovers determine whether the principal should pay the agent more for the signal from the other player ($Pr_j(H|1,1) \lesseqgtr Pr_j(H|1,0)$). Please note that for the case where there is no spillover effect, a range of equilibria exists as listed in Proof for Proposition 1 in the Supplementary Materials. IPE is one of these, and because it is linked to the absence of a spillover effect, this case reverts to the result of the classical principal-agent model with one agent. The level of $I_i(\mathbf{R}_i^*)$ can be interpreted as the informativeness of the signal pair with the highest incentive efficiency: the more informative the signal pair is, the lower the expected payment.

One specific example of asymmetric spillovers is that of the technological department, represented by agent i, and the compliance group, represented by agent j, mentioned earlier in the introduction. The more effort the technological department puts into innovating, the longer the list of objections that the compliance department will raise. Therefore, the technological department should be paid by CPE, as both technological breakthroughs and objections are a signal of the technological department's effort. However, a low level of technological breakthroughs is, at the same time, a signal of *high* effort by the compliance department (along with the number of objections). This is because if the compliance department puts in a high level of effort, it will reduce the number of breakthroughs (where it should be noted that breakthroughs are not always good news to the principal: they are not very useful if they violate numerous regulations). The compliance department should therefore be paid through RPE.[8]

3.4 Comparative statics for expected payment

Apart from being concerned with what signal pair to reward, the principal will also be concerned with the level of the expected payment that needs to be paid to induce effort (1, 1) from agents. As stated in Proposition 1, the expected payment consists of the certain cost of exerting high effort for agents and the incentive efficiency attached with the awarded signal pair(s), which can be rewritten as:

$$\frac{c}{I_i(\mathbf{R}_i^*|1,1)} = \frac{c}{1 - \frac{Pr_i(\mathbf{R}_i^*|0,1)}{Pr_i(\mathbf{R}_i^*|1,1)}} \tag{10}$$

[8]Another example for this asymmetric spillover effect is for the production and the quality control group of a car company. Similarly, a high effort invested by the quality control group would result in more deficiencies, leading to a low signal for the production group. However, a high effort by the production group provides more samples for the quality control group to inspect, potentially leading to a more accurate signal for the quality control group.

Conveniently, expected payment is a positive function of $\frac{Pr_i(\mathbf{R}_i^*|0,1)}{Pr_i(\mathbf{R}_i^*|1,1)}$, so if one knows how this expression changes with the parameters, one is also aware of how the expected payment changes. Note that Assumption 1 (informational independence) and 2 (monotone likelihood ratio property) allow us to disentangle $\frac{Pr_i(\mathbf{R}_i^*|0,1)}{Pr_i(\mathbf{R}_i^*|1,1)}$ further:

$$\frac{Pr_i(\mathbf{R}_i^*|0,1)}{Pr_i(\mathbf{R}_i^*|1,1)} = \frac{Pr_i(R_i^*|0,1)}{Pr_i(R_i^*|1,1)} \cdot \frac{Pr_j(R_j^*|1,0)}{Pr_j(R_j^*|1,1)} \tag{11}$$

In (11), the first term $\frac{Pr_i(R_i^*|0,1)}{Pr_i(R_i^*|1,1)}$ is a relative probability that measures the noisiness of agent i's own signal. To see why, note that as stated in Lemma 2, the optimal signal for agent i (R_i^*) always equals H as $Pr_i(H|0,1) < Pr_i(H|1,1)$. This enables us to rewrite the first term as $\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)}$, the value of which depends on the quality of the signal. The less noise contained within the signal, the lower, therefore, this relative probability $\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)}$. The decrease of this relative probability would lead to an increase of incentive efficiency $I_i(\mathbf{R}_i^*|1,1)$, meaning less cost of inducing a high effort from agent i using signal pair $\mathbf{R}_i^*$ and consequently reducing the cost of extracting effort pair $(1,1)$.[9] Then, regarding the second term, one should define the intensity of the spillover effects to facilitate further discussion on the determinants of these incentive efficiencies.

Definition 4. *The intensity of the spillover effect that agent i has on agent j is defined as:*

$$S_i(1) \equiv \frac{Pr_j(R_j^{i,*}|1,1)}{Pr_j(R_j^{i,*}|1,0)}$$

where $i \neq j \in \{1,2\}$, and $R_j^{i,*}$ is the optimal signal for agent j considered from agent i's perspective.

Spillover intensity measures how much a low effort, rather than a high effort, invested by player i affects the probability that the signal R_j^* is received for player j, given that player j is doing high effort. More straightforwardly, it measures the relative intensity of the spillover effect, whether it is a negative or a positive spillover effect. As suggested in Proposition 1, R_j^* is determined by the ratio $\frac{Pr_j(H|1,1)}{Pr_j(H|1,0)}$. Namely, the principal should select

$$\begin{cases} R_j^{i,*} = H, \text{ if there is a positive spillover effect } (\frac{Pr_j(H|1,1)}{Pr_j(H|1,0)} > 1) \text{ and therefore } S_i(1) > 1 \\ R_j^{i,*} = L, \text{ if there is a negative spillover effect } (\frac{Pr_j(H|1,1)}{Pr_j(H|1,0)} < 1) \text{ and therefore } S_i(1) > 1 \\ R_j^{i,*} = H \text{ or } L, \text{ if there are no spillover effects } (\frac{Pr_j(H|1,1)}{Pr_j(H|1,0)} = 1) \text{ and therefore } S_i(1) = 1 \end{cases}$$

where $R_j^{i,*}$ represents the optimal signal attached with a positive bonus considered from agent i's perspective, and $i \neq j \in \{1,2\}$. One can therefore conclude that $S_i(1) > 1$ as long as spillover

[9] For instance, two similar signals about employees' efforts could be total working hours, and how much time they spent on non-work-related browsing on the Internet. The first signal is noisier because workers may spend their working hours on tasks irrelevant to the outcome.

effects exist, and will equal one if and only if there are no spillover effects. Hence, as $S_i(1)$ equals 1 when there are no spillover effects, and is larger than 1 whether there are positive or negative spillover effects, $S_i(1)$ is a scalar measurement of the intensity of the spillover effect, whether positive or negative. Therefore, the second term of (11) can be rewritten as $\frac{1}{S_i(1)}$, representing the inverse intensity of the spillover effect agent i has on agent j if agent j makes high effort. This would be close to one if the spillover effect is weak and approaching zero if the spillover effect is strong. We can interpret strong spillover effects as another indication that the principal is well-informed, in addition to the low noise in the agent's signal, because the $R_j^* = H$ for positive spillover effects and $R_j^* = L$ for negative spillover effects carry more information about agent i's high effort invested. In conclusion, we have Lemma 3.

Lemma 3. *All else equal, the expected payment ($\frac{c}{I_i(\boldsymbol{R}_i^*|1,1)}$) for inducing effort pair (1,1) for agent i is decreased if 1) the noisiness of agent i's own signal is lower ($\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \downarrow$); 2) The intensity of the spillover effect agent i has on agent j is stronger ($S_i(1) \uparrow$).*

One can interpret the result in terms of the information attached to the signal pair(s). The enlarged spillover effect leads to higher incentive efficiency for the chosen signal pair(s), meaning that the larger the spillover effect, the more information is delivered by receiving a specific signal pair. A similar story can be told for noisiness: the less noisy the signal, the more information it offers to the principal. Consequently, the principal can pay a lower information rent.

4 Optimal effort pair for principals

In the previous analysis, a model where high effort is induced from both agents was considered as a benchmark. However, this setting cannot cover all scenarios. For instance, in the example of a firm with a technological and a marketing department, a high effort from the marketing group would promote the product to the public and increase its adoption rate. Yet, the added revenue from attracting new users might be less than the extra cost of inducing a high effort from the marketing department. In that case, firms are willing to induce only low effort from the marketing group, as it is more financially efficient.

To explore under what circumstances the principal would be willing to induce higher effort from both agents (full efforts), and to analyze the tradeoff between inducing high efforts and having higher revenue but also higher costs, and inducing partial effort and having lower costs but also lower revenue, this section starts by relaxing the assumption that high efforts are always optimal for the principal to induce and introduces a general form of the production function to calculate the expected profit for the principal. Extending the analysis by also considering the case where the principal prefers to induce a partial effort is relevant for start-up firms. They first have to gather a large number of customers before reaching maturity and moving to the next stage of business, where they may only want to induce a partial effort from some of the departments. Note that we refer to an effort pair as full efforts if high effort is induced from both agents, and refer to an effort pair as a partial effort from agent i if a zero effort is induced from agent i and a high effort is induced from agent j. Specifically, subsection 4.1 first extends optimal incentive schemes for effort pairs other than (1,1). Subsection 4.2 then provides comparative-statics results by studying the effect of changes in the noisiness of signals and in the intensity of the spillover effects on both added revenue and extra cost of inducing full efforts. This serves as a preparation for finding the optimal effort pair. Subsection 4.3 closes by providing sufficient conditions for optimality of full efforts, and pinpointing how these depend on the noisiness of signals and on the intensity of the spillover effects.

4.1 Optimal incentive schemes for given effort pair (e_i^*, e_j^*)

To maintain consistency within the framework, one should aim to extend Proposition 1 from exerting effort levels $(e_i^*, e_j^*) = (1, 1)$ to any effort pair (e_i^*, e_j^*). Start with the extension of the definition of incentive efficiencies. Specifically:

Definition 5. *For agent i at signal pair $\boldsymbol{R}_i$ with optimal effort pair (e_i^*, e_j^*), information efficiency ($I_i(\boldsymbol{R}_i|e_i^*, e_j^*)$) is defined as*

$$I_i(\boldsymbol{R}_i|e_i^*, e_j^*) \equiv \frac{Pr_i(\boldsymbol{R}_i|e_i^*, e_j^*) - Pr_i(\boldsymbol{R}_i|e_i^{-*}, e_j^*)}{Pr_i(\boldsymbol{R}_i|e_i^*, e_j^*)} \text{ for } \forall i = 1, 2$$

(e_i^*, e_j^*) represents the optimal effort pair induced by the principal, and $e_i^{-*} \neq e_i^*$ represents an effort level other than e_i^*.

To include the case where the principal wants to induce zero effort from one of the agents, we extend the MLRP assumption.

Assumption 3. *For $\forall i \in \{1, 2\}$, the probability of receiving an H signal obeys the following inequality:*

$$Pr_i(H|1, e_j) > Pr_i(H|0, e_j) \tag{12}$$

which is equivalent to the monotone likelihood ratio property under Assumption 1, where $e_j \in \{0, 1\}$.

This means that high effort from one agent always leads to a higher probability of receiving an H signal. Besides, the definition of positive/negative spillover effects needs to be extended for the effort pair (e_i, e_j) as follows:

Definition 6. *A positive spillover effect of agent i on agent j means*

$$Pr_j(H|e_j, 1) > Pr_j(H|e_j, 0) \tag{13}$$

A negative spillover effect of agent i on agent j means

$$Pr_j(H|e_j, 1) < Pr_j(H|e_j, 0) \tag{14}$$

No spillover effect from agent i to agent j means

$$Pr_j(R_j|e_j, 1) = Pr_j(R_j|e_j, 0),\ for\ \forall R_i \in [L, H] \tag{15}$$

where $e_j \in \{H, L\}$.

This shows that Definition 2 is a special case of Definition 6 where one takes $e_j = 1$. Following the definition, one can interpret the positive spillover effect in (13) when $e_j = 0$ in the following way: taking agent i as the technological department and agent j as the marketing department, high sales (as such a signal of high effort of the marketing department) is a signal of high effort by the technological department as well, even if the marketing department made low effort: these high sales could result from word of mouth. Namely, if the technological department puts in high effort rather than low effort, then one is more likely to obtain a positive signal in the form of high sales from the marketing department.

Leaving the participation constraints and limited liability constraints in the same form as before, the introduction of a given effort pair (e_i^*, e_j^*) alters the incentive constraint into

$$\{E_{\mathfrak{R}_i}[\omega_i(\mathbf{R}_i)|1, e_j^*] - E_{\mathfrak{R}_i}[\omega_i(\mathbf{R}_i)|0, e_j^*] - c\} \cdot (-1)^{\mathbb{1}_{e_i^*=0}} \geq 0 \text{ for } \forall i = 1, 2 \tag{16}$$

where $\mathbb{1}_{e_i^*=0}$ is an indicator function which equals one if $e_i^* = 0$, and zero otherwise. Similarly, the participation constraint is also altered accordingly based on whether or not $e_i^* = 1$

$$E_{\mathfrak{R}_i}[\omega_i(\mathbf{R}_i)|e_i^*, e_j^*] - c \cdot \mathbb{1}_{e_i^*=1} \geq 0 \text{ for } \forall i = 1, 2. \tag{17}$$

Then, one can extend Proposition 1 to calculate the optimal incentive scheme as follows, given that the principal wants to extract the effort pair (e_i^*, e_j^*).

Proposition 2. *Under informational independence (Assumption 1), and given the monotone likelihood ratio property (Assumption 3):*

- *if* $e_i^* = 0$*, the optimal payment entails*

$$\omega_i^*(\boldsymbol{R}_i) = 0 \, for \, \forall \boldsymbol{R}_i \in \Re_i^2$$

with expected payment equal to zero.

- *if* $e_i^* = 1$*, the optimal payment entails*

  (i) Collective Performance Evaluation ($\omega_i^*(H,H) > \omega_i^*(H,L) = 0$*), if agent i has a positive spillover effect on agent j (*$Pr_j(H|e_j^*, 1) > Pr_j(H|e_j^*, 0)$*).*

  (ii) Relative Performance Evaluation ($\omega_i^*(H,L) > \omega_i^*(H,H) = 0$*), if agent i has a negative spillover effect on agent j (*$Pr_j(H|e_j^*, 1) < Pr_j(H|e_j^*, 0)$*).*

  (iii) Independent Performance Evaluation ($\omega_i^*(H,L) = \omega_i^*(H,H) > 0$*), if agent i has no spillover effect on agent j (*$Pr_j(H|e_j^*, 1) = Pr_j(H|e_j^*, 0)$*).*[10]

  Specifically, the optimal incentive scheme take the form:

$$\begin{cases} \omega_i^*(\boldsymbol{R}_i^*) = \frac{c}{Pr_i(\boldsymbol{R}_i^*|1,e_j^*) - Pr_i(\boldsymbol{R}_i^*|0,e_j^*)} \\ \omega_i^*(\boldsymbol{R}_i^{-*}) = 0 \end{cases}$$

  with the expected payment equal to $\frac{c}{I_i(\boldsymbol{R}_i^*|1,e_j^*)}$*.*

$\boldsymbol{R}_i^*$ *represents the signal pair(s) attached with the highest incentive efficiency* $I_i(\boldsymbol{R}_i^*|1, e_j^*)$*, which is determined by the inequality in (i), (ii), and (iii), and* $\boldsymbol{R}_i^{-*}$ *represents any signal pair other than* $\boldsymbol{R}_i^*$*.*[11]

Proposition 2, serving as an extension of Proposition 1, provides the optimal incentive scheme for any given effort pair $(e_i^*, e_j^*) \in \Re^2$. Specifically, in the case where the principal wants to induce zero effort from an agent ($e_i^* = 0, e_j^* = 0, 1$), the principal should pay him zero, regardless of which signal pair is ultimately reached, by making the agent's participation constraints binding. Moreover, apart from the case where the principal wants to induce ($e_i^* = 1, e_j^* = 1$), which has already been analyzed in Proposition 1, the case where the principal is willing to induce ($e_i^* = 1, e_j^* = 0$) for $i = 1, 2$ serves as a direct extension of it, where CPE is optimal if agent i has a positive spillover effect on agent j ($Pr_j(H|e_i^*, 0) < Pr_j(H|e_i^*, 1)$, for all $e_i^* \in \{0, 1\}$), and RPE is optimal if agent i has a negative spillover effect on agent j ($Pr_j(H|e_i^*, 0) > Pr_j(H|e_i^*, 1)$, for all $e_i^* \in \{0, 1\}$). For instance, with i the compliance department and j the technological department, the compliance department can

[10] Note that without spillovers, if one incentive efficiency is weakly highest, the indicated payment is one of several optimal payments, as shown in the proof.

[11] Note that we can also apply $c \cdot \frac{\mathbb{1}_{e_j^*=1}}{I_j(\mathbf{R}_j^*|1,e_i^*)}$ as the general form for expected payment for $\forall e_i = 0, 1$.

have a negative spillover effect on the technological department even if the technological department makes a low effort: a high signal (in the form of technological breakthroughs, such as the number of new features deployed) continues to be possible then even if the technological department invest a low effort, but again its probability is reduced if the compliance group makes high effort in reporting violations. Under these circumstances, the principal would only pay the compliance department a positive bonus if signal pair $\mathbf{R}_j = (H, L)$ is achieved, which is the same as the signal pair when full efforts is induced and analyzed in subsection 3.4. However, the expected cost of inducing full efforts or a partial effort can be different, as discussed in the following subsections.

4.2 Comparative-statics for expected profit

To enable the comparison between the expected payoff of inducing full efforts and the expected payoff of inducing a partial effort, one should consider the principal's profit function first. Assume that each effort pair and/or signal pair generates a production level, which is sold at a normalized price, where the function relating signal pairs and/or effort pairs to production levels can be thought of as a production function. Set the principal's profit function ($\pi(\cdot)$) as the expected revenue generated minus the expected payment for both agents, where expected revenues equal the expected production of agents ($\sum_{\mathbf{R}_i \in \mathfrak{R}_i^2} F_i(\mathbf{R}_i, e_i, e_j) Pr_i(\mathbf{R}_i, e_i, e_j)$) multiplied by price normalized to 1, where $F_i(\mathbf{R}_i, e_i, e_j)$ is a general form of the production function. Note that for simplicity, we introduce the following assumption concerning zero effort pairs from both agents:

Assumption 4. *The principal never prefers to induce zero effort from both agents.*

Hence, Assumption 4 helps to narrow down the potential desired effort pair to either full efforts or partial effort. Then, one can check whether it is optimal to induce high efforts from both agents by comparing the principal's maximized profit associated with the desired effort pair. Therefore, the expected profit for the principal inducing effort pair (e_i^*, e_j^*) equals

$$E_{\mathfrak{R}_i}(\pi | e_i^*, e_j^*) = \sum_{k=1}^{4} F_i(\mathbf{R}_{k,i}, e_i^*, e_j^*) Pr_i(\mathbf{R}_{k,i} | e_i^*, e_j^*) - c \left\{ \frac{\mathbb{1}_{e_i^*=1}}{I_i(\mathbf{R}_i^* | 1, e_j^*)} + \frac{\mathbb{1}_{e_j^*=1}}{I_j(\mathbf{R}_j^* | 1, e_i^*)} \right\} \tag{18}$$

where $\mathbf{R}_{k,i}$ represents signal pairs from agent i's perspective, meaning the first effort is from agent i and the other effort is from agent j. This can be specified into $\mathbf{R}_{1,i} = (H, H)$, $\mathbf{R}_{2,i} = (H, L)$, $\mathbf{R}_{3,i} = (L, H)$, $\mathbf{R}_{4,i} = (L, L)$, which lists all possible signal pairs the principal might receive, indexed from signal pair 1 to 4. $\mathbb{1}_{e_i^*=1}$ is an indicator function that equals one if $e_i^* = 1$ and zero otherwise. The signal pair with the highest incentive efficiency $(R_i^{i,*}, R_j^{i,*})$ can be different from $(R_i^{j,*}, R_j^{j,*})$ if two agents have asymmetric spillover effects. Without loss of generality, we compare inducing effort (1,1) to inducing effort (0,1) - so it is agent i that is considered as possibly doing zero effort, where it is optimal for the principal to induce full efforts if and only if the change of expected profit is positive ($E_{\mathfrak{R}_i}(\pi|1,1) \geq E_{\mathfrak{R}_i}(\pi|0,1)$):

$$\begin{aligned}&\{\sum_{k=1}^{4} F_i(\mathbf{R}_{k,i},1,1)Pr_i(\mathbf{R}_{k,i}|1,1) - \sum_{k=1}^{4} F_i(\mathbf{R}_{k,i},0,1)Pr_i(\mathbf{R}_{k,i}|0,1)\}\\&- c \cdot \left\{\frac{1}{I_i(\mathbf{R}_i^*|1,1)} + \frac{1}{I_j(\mathbf{R}_j^*|1,1)} - \frac{1}{I_j(\mathbf{R}_j^*|1,0)} - 0\right\} \geq 0.\end{aligned} \tag{19}$$

For simplicity, we refer to (19) as the change in expected profit, where the first term represents the added revenue, and the second term represents the extra cost of inducing full efforts compared to inducing a partial effort from agent i. Thus, this comparison boils down to the tradeoff between the added revenue the principal can earn by extracting full efforts and its extra cost, and whether it is optimal to induce high effort depends on whether the added revenue exceeds the extra cost. We therefore begin our analysis by exploring how the intensity of the spillover effects and the noisiness of the signals affect the extra cost (Section 4.2.1) and the added revenue (Section 4.2.2).

4.2.1 Comparative statics for extra cost

Given our assumption that an agent's cost of doing full effort is c and his cost of doing partial effort is 0, the extra cost is then controlled by inverse incentive efficiencies $\frac{1}{I_i(\mathbf{R}_i^*|1,1)}$ and $\frac{1}{I_j(\mathbf{R}_j^*|1,e_i)}$:

$$\frac{1}{I_i(\mathbf{R}_i^*|1,1)} + \frac{1}{I_j(\mathbf{R}_j^*|1,1)} - 0 - \frac{1}{I_j(\mathbf{R}_j^*|1,0)} \tag{20}$$

where the first two terms represent the expected payment when inducing full efforts (1,1), whereas the last two terms represent the expected payment when inducing a partial effort from agent i ($e_i = 0, e_j = 1$). Note that each term in (20), except zero, shares the same structure ($\frac{1}{I_i(\mathbf{R}_i^*|e_i,1)}$), which is an extension of (10), but where an effort pair other than (1, 1) could be induced. It is straightforward that a decrease in the cost of inducing full efforts, and an increase in the cost of inducing a partial effort from agent i will reduce the extra cost and consequently make the principal more willing to exert full efforts.

To explore the determinants of these inverse incentive efficiencies, we first update Definition 4 to conduct a similar analysis as in subsection 3.4 to disentangle ($\frac{1}{I_i(\mathbf{R}_i^*|1,1)}$) into two components: noisiness of signals and intensity of spillover effects. Therefore, $\frac{1}{I_i(\mathbf{R}_i^*|e_i^*,1)}$ has the following determinants:

$$\frac{Pr_i(R_i^*|e_i^{-*},1)}{Pr_i(R_i^*|e_i^*,1)} \cdot \frac{Pr_j(R_j^{i,*}|1,e_i^{-*})}{Pr_j(R_j^{i,*}|1,e_i^*)} \tag{21}$$

where $R_j^{i,*}$ is the optimal signal for agent j considered from agent i's perspective. Similarly, we extend Definition 4 of the spillover effect also to a partial effort case.

Definition 7. *The intensity of the spillover effect that agent i has on agent j is defined as:*

$$S_i(e_j) \equiv \frac{Pr_j(R_j^{i,*}|e_j, 1)}{Pr_j(R_j^{i,*}|e_j, 0)}$$

where $e_j \in \{0, 1\}$, and $R_j^{i,*}$ is the agent j's signal that should be rewarded with a positive payment for agent i.

Hence, following the same line of reasoning as in Lemma 3, we can extend its result to Lemma 4.

Lemma 4. *All else equal, the expected cost for agent i when the principal induces effort (e_i^*, e_j^*) is decreased if:*

(i) the spillover effect agent i has on agent j is more intense ($\frac{Pr_j(R_j^{i,*}|e_j^*,1)}{Pr_j(R_j^{i,*}|e_j^*,0)} \uparrow$).

(ii) the noisiness of agent i's own signal is lower ($\frac{Pr_i(H|0,e_j^*)}{Pr_i(H|1,e_j^*)} \downarrow$).

Namely, a reduction in noisiness and an increase in the intensity of the spillover effects decrease the expected cost, as this indicates that the signal pair $\mathbf{R}_i^*$ reveals more information and limits the effect of the asymmetric information between agents and the principal. Then, it is straightforward that the extra cost will be lower if the expected payment of inducing a partial effort is high ($\frac{1}{I_j(\mathbf{R}_j^*|1,0)} \uparrow$) and the cost of inducing (1,1) is low ($\frac{1}{I_j(\mathbf{R}_j^*|1,1)} \downarrow$ and $\frac{1}{I_i(\mathbf{R}_i^*|1,1)} \downarrow$).

Hence, as a direct corollary of Lemma 4, Corollary 4.1 states conditions under which the principal is, from a cost perspective, more willing to exert full efforts, all else equal.

Corollary 4.1. *All else equal, the change in extra cost (19) decreases*

(i) the more intense the agents' spillover effect when full efforts are induced ($S_i(e_j^* = 1) \uparrow$ and $S_j(e_i^* = 1) \uparrow$), and the less intense agent j's spillover effect when a partial effort is induced ($S_j(e_i^* = 0) \downarrow$).

(ii) the noisier agent j's signal if agent i invests a zero effort ($\frac{Pr_j(H|0,0)}{Pr_j(H|1,0)} \uparrow$), and the less noisy agent i's own signal if the other agent invests effort one ($\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \downarrow$).

In other words, the extra cost is lower, and the principal will tend more to induce full efforts rather than a partial effort from agent i, the smaller the spillover effect from agent j on agent i if the principal induces a partial effort from agent i, and the larger the spillover effects from both agents if full efforts is induced from them. The same occurs when the noisiness of the agents' signals is smaller when full efforts is induced from them, and when the noisiness of agent j's signal is larger when a partial effort is induced from agent i.

4.2.2 Comparative statics for added revenue

Next, we move to the comparative statics for added revenue, which is the other component that affects (19). To further compare the change in added revenue, one indispensable element is the function of the signals. Does the firm only care about the efforts, with H, L purely signals of efforts? Or are H and L stochastic production levels generated by the efforts, in which case they are, at the same time, signals of the efforts as well? For instance, if the agent works from home, the principal may use the agent's response time to messages as a signal for actual working hours. If hours worked now directly translate into revenue for the firm (without any uncertainty involved), so that the firm cares only about working hours, we have the case where signals H and L are purely signals of effort. However, if there is additionally uncertainty involved, in that efforts translate into production levels stochastically, then the production levels serve as a signal of the agents' efforts and are also, at the same time, what the firm values (H, L are production levels). From a mathematical perspective, the former case leads to $F_i(\mathbf{R}_{k,i}, e_i, e_j) = F_i(e_i, e_j)$. In contrast, the latter case leads to $F_i(\mathbf{R}_{k,i}, e_i, e_j) = F_i(\mathbf{R}_{k,i})$. As we focus on the first term in (19), the comparison when the firm only cares about the effort is straightforward since it is purely the difference between the production levels for full efforts and a partial effort:

$$F_i(1, 1) - F_i(0, 1) \tag{22}$$

Under this setting, spillover effects or noise within signals have no impact on added revenue, as the added revenue depends solely on the shape of the production function. In this case, the full comparative statics of the profit function depend on Corollary 4.1.

We further proceed with our analysis of the second case, where H, L are not only signals, but also enter the production function. We assume for this case that $F_i(H, s) > F_i(L, s)$, and $F_i(s, H) > F_i(s, L)$, for $s = L, H$, namely, the higher the obtained production level (and the higher the signal), the larger the profit obtained. Under this setting, after applying informational independence (Assumption 1), $E_{\mathfrak{R}_i}(\mathbf{R}_i|e_i, e_j)$ can be derived as:

$$\begin{aligned}
E_{\mathfrak{R}_i}(\mathbf{R}_i|e_i, e_j) &= \sum_{k=1}^{4} Pr_i(\mathbf{R}_i|e_i, e_j)F_i(\mathbf{R}_i) \\
&= \sum_{k=1}^{4} Pr_i(R_i|e_i, e_j)Pr_j(R_j|e_j, e_i)F_i(\mathbf{R}_i) \\
&= Pr_i(H|e_i, e_j)Pr_j(H|e_j, e_i)[F_i(H, H) - F_i(L, H) - F_i(H, L) + F_i(L, L)] \\
&\quad + Pr_i(H|e_i, e_j)[F_i(H, L) - F_i(L, L)] + Pr_j(H|e_j, e_i)[F_i(L, H) - F_i(L, L)] + F_i(L, L)
\end{aligned} \tag{23}$$

One can easily observe that the expected revenue, given the effort pair (e_i, e_j), depends on the shape of the production function and of the probability functions, which are determined by the noisiness of signals and by the spillover effects. The following definition will be useful to describe comparative statics on added revenue:

Definition 8. *The productive cross difference is defined as:*

$$[F_i(H,H) - F_i(L,H)] - [F_i(H,L) - F_i(L,L)] \tag{24}$$

We say that the production function is characterized by productive complementarity if (24) is positive, by productive substitutability if (24) is negative, and by neither of these if (24) is zero.[12] For simplicity, we have the following Assumption 5:

Assumption 5. *The production function has a constant margin between signals*

$$F_i(H,H) - F_i(H,L) = F_i(L,H) - F_i(L,L) = \alpha > 0. \tag{25}$$

This means that there is neither productive complementarity nor productive substitutability ($[F_i(H,H) - F_i(L,H) - F_i(H,L) + F_i(L,L)] = 0$). The purpose of this assumption is to disentangle the effect of the production function and purely emphasize the impact of probabilities on the selection of the optimal effort pair. Then $E_{\mathfrak{R}_i}(\mathbf{R}_i|e_i,e_j)$ can be written as:

$$E_{\mathfrak{R}_i}(\mathbf{R}_i|e_i,e_j) = \alpha[Pr_i(H|e_i,e_j) + Pr_j(H|e_j,e_i)] + F_i(L,L)$$

and hence the added revenue equals:

$$E_{\mathfrak{R}_i}(\mathbf{R}_i|1,1) - E_{\mathfrak{R}_i}(\mathbf{R}_i|0,1) = \alpha[Pr_i(H|1,1) - Pr_i(H|0,1)] + \alpha[Pr_j(H|1,1) - Pr_j(H|1,0)] \tag{26}$$

Following the same line of reasoning as in subsection 3.4, the first term of (26) is the noisiness of agent i's signal, and the second term is the intensity of the spillover effect of agent i on agent j, also expressed in absolute terms, where the spillover effect may be positive or negative, but where these are now measured in absolute terms. Note that $Pr_j(H|1,1) - Pr_j(H|1,0)$ is positive if agent i has a positive spillover effect on agent j and negative if it has a negative spillover effect instead. Then, we obtain Lemma 5:

Lemma 5. *With a constant margin between signals for the production function (Assumption 5), all else equal, the added revenue from inducing full efforts rather than a partial effort (19) increases:*

[12] What we call productive complementarity is also referred to as supermodularity, and what we call productive substitutability is also referred to as submodularity(Topkis, 1998).

(i) *the more intense the positive spillover effect is from agent i ($S_i(e_j = 1) \uparrow$) on agent j, and the less intense the negative spillover effect is from agent i on agent j in absolute terms ($Pr_j(H|1,1) - Pr_j(H|1,0) \uparrow$).*

(ii) *the less noisy agent's i's signal is ($\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \downarrow$) if effort 1 is induced from agent j.*

Note that the increase in intensity of the positive spillover effect $S_i(e_j = 1)$ leads to a decrease of $Pr_j(H|1,1) - Pr_j(H|1,0)$ and consequently to a decrease in added revenue, whereas an increase in the intensity of the negative spillover effect would have an opposite effect on $Pr_j(H|1,1) - Pr_j(H|1,0)$, which leads to a increase of it. For consistency, we express the results in terms of relative intensity or relative noisiness whenever possible, and use absolute terms only when additional conditions are necessary.

Lemma 5 shows that a positive spillover effect from agent i increases the principal's willingness to induce full efforts; however, a negative spillover effect from agent j has the opposite effect, which increases the willingness to induce a partial effort as it leads to lower added revenue. Additionally, less noise contained within agent i's signal also increases the added revenue.

As a conclusion for the entire Section 4.2, by combining the comparative statics for the extra cost in Corollary 4.1 and the comparative statics for the added revenue in Lemma 5, we obtain in Proposition 3 the following comparative statics results for the change in expected profit:

Proposition 3. *With a constant margin for production function (Assumption 5), all else equal, the change of expected profit from inducing full rather than a partial effort (19) increases:*

(i) *the more relatively intense the agents' spillover effect when effort 1 is induced ($S_i(e_j^* = 1) \uparrow$ and $S_j(e_i^* = 1) \uparrow$), and the less relatively intense agent j's spillover effect when effort 0 is induced ($S_j(e_i^* = 0) \downarrow$).*

(ii) *the less intense the spillover effect is from agent i in absolute terms ($Pr_j(H|1,0) - Pr_j(H|1,1) \downarrow$) if agent i has a negative spillover effect.*

(iii) *the noisier agent j's signal if agent i invests a zero effort ($\frac{Pr_j(H|0,0)}{Pr_j(H|1,0)} \uparrow$), and the less noisy agent i's own signal if the other agent invests effort 1 ($\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \downarrow$).*

(iv) *the lower the cost of investing effort c for agents ($c \downarrow$).*

Proposition 3 shows that spillover effects and the noisiness of signals jointly determine the principal's choice between inducing full and partial efforts. Regarding the intuition behind spillover effects in the same direction, consider the marketing department as agent i and the technological department as agent j. The principal decides whether to induce a partial effort from the marketing department, who is more incentivized to induce full efforts if (Proposition 3(i)): the two departments reinforce each other mainly under full efforts. Intuitively, intensive marketing is more valuable when it is to a larger extent the case that a better product gives the marketing department more persuasive material to promote ($S_i(e_j^* = 1) \uparrow$), and when it is to a larger extent the case that strong marketing creates more users, feedback, and market visibility for the product and hence speeds up the technological advancement ($S_j(e_i^* = 1) \uparrow$); Also, intensive marketing is more valuable when it is to a larger extent the case that low effort from the marketing department blocks technological advancement ($S_j(e_i^* = 0) \downarrow$).

For spillover effects in the opposite direction, we rather take the compliance department as agent i. On top of the condition described in the previous example, the principal is more incentivized to induce full efforts if (Proposition 3(ii)): the scrutiny conducted by the compliance department has less impact on the new features deployed by the technological department if the latter exerts high effort ($Pr_j(H|1,1) - Pr_j(H|1,0) \uparrow$). Combined with Proposition 3(i), this means high compliance effort creates a strong signal for contracting, while causing only a limited reduction in actual technological output.

We next look at the effect of the noisiness of the signals. We go back to the example where the marketing department is agent i and the technological department is agent j. The principal is more incentivized to induce full effort if (Proposition 3(iii)): when the marketing department induces zero effort, and the technological department does high rather than low effort, a low signal from the technological department is more often obtained; In other words, the principal should induce high effort from the marketing department if, when instead inducing low effort from the marketing department, it becomes more difficult to conclude that a low signal from the technological department is due to this department's low effort, as it may be due to the marketing department's low effort ($\frac{Pr_j(L|0,0)}{Pr_j(L|1,0)} \downarrow$ becomes smaller, meaning $\frac{Pr_j(H|0,0)}{Pr_j(H|1,0)} \uparrow$); Also, on the other hand, the principal is more incentivized to induce full effort if, in case high effort is induced from the technology department, when the marketing department does high effort rather than low effort, a high signal from the marketing department is more often obtained. In other words, when the technological department puts in high effort, it becomes easier to conclude from a high signal (e.g., large sales) of the marketing department that the marketing department did high rather than low effort. In other words, with low effort from the technological department, the marketing department may more often fail even if it did high effort ($\frac{Pr_j(L|0,1)}{Pr_j(L|1,1)} \uparrow$ becomes larger, meaning $\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \downarrow$).

In general, Proposition 3 hence suggests that 1) intensity of spillover effects, 2) noisiness of signals, and 3) direction of spillover effects jointly determine whether the principal should induce full or partial efforts.

4.3 Sufficient conditions for optimal effort pairs

Optimal incentive schemes (4.1) and what factors increase or decrease the change in expected profits from inducing high effort have already been discussed using comparative statics (4.2). However, note that these comparative-static results do not directly tell the principal whether inducing full efforts or a partial effort is optimal, since the expected profit can remain negative (positive) even after we have increased (decreased) it. Instead, it depends on whether the added revenue exceeds the extra cost. To obtain an indication of the optimal effort pair and its link to spillover effects/noisiness, this subsection begins with sufficient conditions for optimal effort pairs determined by the shape of the revenue and cost function (4.3.1), and then we explore the links of spillover effects/noisiness to these conditions from both the cost side (4.3.2) and the revenue side (4.3.3) by introducing the concepts of probabilistic difference and revenue cross difference. By combining these two parts of the analysis, we arrive at the final proposition, which directly states the sufficient condition to reach optimal effort pairs in terms of spillover effects/noisiness (4.3.4).[13]

4.3.1 Sufficient conditions for optimal effort pairs for revenue and cost functions

To derive sufficient conditions under which inducing full efforts or a partial effort is optimal, we apply a different perspective by comparing the extra cost of inducing (1,1) rather than (0,1), to the extra cost of inducing (0,1) rather than (0,0). We interpret this in terms of the concavity and convexity of the expected cost: assuming added revenue is linear, if the extra cost of inducing effort (1,1) rather than (0,1) is smaller than the extra cost of inducing (0,1) rather than (0,0), i.e. if the expected cost is concave, then there is a larger range of added revenues under which the principal would not only move from inducing (0,0) to (0,1), but would also move from inducing (0,1) to (1,1); if on the contrary the extra cost of inducing effort (1,1) rather than (0,1) is larger than the extra cost of inducing (0,1) rather than (0,0), i.e. the expected cost is convex, then there is a larger range of added revenues under which the principal would stop at (0,1) and would not proceed further to (1,1).

Hence, as Assumption 4 excludes inducing zero effort from both agents from the desired effort pair, we can reach Lemma 6 for sufficient conditions of optimal effort pairs for cost and revenue functions.

Lemma 6. *After assuming zero effort from both agents is suboptimal (Assumption 4), it is optimal for the principal to induce full efforts*

- *if expected cost is concave, and expected revenue is either convex or linear,*
- *or if expected cost is linear, and expected revenue is convex.*

[13] Note that if the principal is indifferent between inducing full efforts or a partial effort, we assume that she always prefers the full efforts.

The sufficient conditions for an optimal effort pair therefore consist of two parts: the concavity or convexity of costs, and of the revenue function. Note that when these sufficient conditions are not met, then it is possible to induce a partial effort as the optimal choice. Hence, to examine the impact of spillover effects and noisiness on optimal effort pairs, we first explore how they are linked to the convexity and concavity of the cost function.

4.3.2 Change within expected costs and its link to the optimal effort pair

Derived from (20) with additional elements for the extra cost of inducing a partial effort rather than zero efforts, the following equation determines whether the expected cost is convex, concave, or linear depending on whether the equation is larger, smaller, or equal to zero:

$$[\frac{c}{I_i(\mathbf{R}_i^*|1,1)} + \frac{c}{I_j(\mathbf{R}_j^*|1,1)} - \frac{c}{I_j(\mathbf{R}_j^*|1,0)}] - [\frac{c}{I_j(\mathbf{R}_j^*|1,0)} - 0] \lesseqgtr 0. \tag{27}$$

The first part in (27) is the difference in cost, when inducing high effort from agent j, between inducing high or low effort from agent i. The second part is the difference, when inducing zero effort from agent i, between inducing high effort or zero effort from agent j. To interpret (27) in a better way, we introduce the following simplifying assumption.

Assumption 6. *When inducing effort 0 from agent i and effort 1 from agent j, incentive efficiency is the same for both agents:*

$$I_j(\boldsymbol{R}_j^*|1,0) = I_i(\boldsymbol{R}_i^*|1,0)$$

Assumption 6 assumes symmetry between incentive efficiencies for simplicity. However, this does not necessarily lead to homogeneous spillover effects. It is possible to have heterogeneous spillover effects while still satisfying Assumption 6. Under Assumption 6, the cross difference between the extra costs to induce effort (1,1) from (0,1) and (0,1) from (0,0) can be re-expressed as

$$[\frac{c}{I_i(\mathbf{R}_i^*|1,1)} - \frac{c}{I_i(\mathbf{R}_i^*|1,0)}] + [\frac{c}{I_j(\mathbf{R}_j^*|1,1)} - \frac{c}{I_j(\mathbf{R}_j^*|1,0)}], \tag{28}$$

which can be interpreted as the sum of the difference between the expected payment for one agent when the other agent exerts a high effort and a zero effort ($\sum_{j=1,2}[\frac{c}{I_j(\mathbf{R}_j^*|1,1)} - \frac{c}{I_j(\mathbf{R}_j^*|1,0)}]$). Therefore, the question is to further discover the relation between $\frac{c}{I_j(\mathbf{R}_j^*|1,1)}$ and $\frac{c}{I_j(\mathbf{R}_j^*|1,0)}$ for $j = 1, 2$. Note that, similar to the approach adopted in subsection 4.2, we can separate $\frac{c}{I_j(\mathbf{R}_j^*|e_i^*,e_j^*)}$ into two parts: the noise of one's own signal and the intensity of the spillover effect. To specify the sign of (28), we focus on a zero cross difference for the probability function as assumed in Assumption 7, which allows us to focus on the effect of the form of the production function.

Assumption 7. *The probability function has zero cross difference*

$$[Pr_i(H|1,1) - Pr_i(H|1,0)] - [Pr_i(H|0,1) - Pr_i(H|0,0)] = 0 \tag{29}$$

Note that (29), as a cross difference for the probability function, can be interpreted either as extra conditions on the intensity of the spillover effect or on the signal's noisiness. On the one hand, $[Pr_i(H|1,1) - Pr_i(H|1,0)]$ and $[Pr_i(H|0,1) - Pr_i(H|0,0)]$ are the absolute intensity of spillover effects $S_i(e_j = 1)$ and $S_i(e_j = 0)$ as a scalar value rather than in ratios.[14] On the other hand, (29) can also be interpreted from a noisiness perspective if we change the order of $Pr_i(H|1,0)$ and $Pr_i(H|0,1)$. The first part is then the reduced noisiness within agent i's signal from exerting a high rather than a low effort if agent j exerts a high effort ($[Pr_i(H|1,1) - Pr_i(H|0,1)]$), expressed in absolute terms, and the second part is the reduced noisiness within agent i's signal from exerting a high effort rather than a low effort if agent j exerts a zero effort ($[Pr_i(H|1,0) - Pr_i(H|0,0)]$). Zero probabilistic cross difference indicates that effort of the other agent neither increases nor decreases the marginal probability of reaching a high signal in absolute terms. Equipped with Assumption 7, we reach Lemma 7 for specifying the relation between the form of the cost function and the relation between the spillover effects.

Lemma 7. *With symmetric incentive efficiency for a partial effort (Assumption 6) and zero cross difference for the probability function (7), for agent i, the cost function*

- *is convex* ($\frac{c}{I_j(\boldsymbol{R}_j^*|1,1)} < \frac{c}{I_j(\boldsymbol{R}_j^*|1,0)}$) *if the spillover effect of agent j on agent i is more intense when i does high effort than when i does low effort* ($S_j(e_i = 1) > S_j(e_i = 0)$).
- *is concave* ($\frac{c}{I_j(\boldsymbol{R}_j^*|1,1)} > \frac{c}{I_j(\boldsymbol{R}_j^*|1,0)}$) *if the spillover effect of agent j on agent i is more intense when i does low effort than when i does high effort* ($S_j(e_i = 1) < S_j(e_i = 0)$).
- *is linear* ($\frac{c}{I_j(\boldsymbol{R}_j^*|1,1)} = \frac{c}{I_j(\boldsymbol{R}_j^*|1,0)}$) *if the spillover effect of agent j on agent i is the same when i does low effort and when i does high effort* ($S_j(e_i = 1) = S_j(e_i = 0)$).

To interpret the result from Lemma 7, it is useful to define the cross difference in terms of log-probabilities, which then serves as a sufficient and necessary condition for determining inequality in spillover effects ($S_j(e_i = 1)$ and $S_j(e_i = 0)$).

Definition 9. *The log probabilistic cross difference is defined as:*

$$[\log(Pr_i(H|1,1)) - \log(Pr_i(H|1,0))] - [\log(Pr_i(H|0,1)) - \log(Pr_i(H|0,0))] \tag{30}$$

[14] Please note that the $S_i(1)$ and $S_i(0)$ can still vary even after zero cross difference for the probability function is assumed, suggesting Assumption 7 does not constrain the inequality between $S_i(e_j = 1)$ and $S_i(e_j = 0)$. For instance, $Pr_i(H|1,1) = 0.8$, $Pr_i(H|1,0) = 0.7$, $Pr_i(H|0,1) = 0.6$, $Pr_i(H|0,0) = 0.5$. Then $Pr_i(R_i|1,1) - Pr_i(R_i|1,0) = Pr_i(R_i|0,1) - Pr_i(R_i|0,0) = 0.1$, and $\frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} = \frac{8}{9} > \frac{Pr_i(H|0,0)}{Pr_i(H|1,0)} = \frac{6}{7}$. One can easily switch these numbers to achieve other scenarios.

for $i \neq j \in \{1,2\}$.

The first term $\log(\frac{Pr_i(H|1,1)}{Pr_i(H|1,0)}) = \log(S_i(e_j = 1))$ and the second term $\log(\frac{Pr_i(H|0,1)}{Pr_i(H|0,0)}) = \log(S_i(e_j = 0))$ are the logarithms of the intensity of positive spillover effects of agent j on agent i if agent i exerts an effort 1 respectively an effort 0. Therefore, (30) draws a direct connection with Lemma 7 for convexity/concavity of the cost function.[15] We now say that efforts are log probabilistic complements, respectively substitutes, in the following cases: if (30) is positive, then two efforts are log probabilistic complements; if it is negative, then two efforts are log probabilistically substitutable, and they are neither log probabilistic complements nor substitutes (linear) if it equals zero.[16] Specifically,

$$\begin{array}{ll} \text{Log probabilistically complementary:} & \frac{Pr_i(H|1,1)}{Pr_i(H|1,0)} > \frac{Pr_i(H|0,1)}{Pr_i(H|0,0)} \text{ or } \frac{Pr_i(H|0,0)}{Pr_i(H|1,0)} > \frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \\ \text{Log probabilistically substitutable:} & \frac{Pr_i(H|1,1)}{Pr_i(H|1,0)} < \frac{Pr_i(H|0,1)}{Pr_i(H|0,0)} \text{ or } \frac{Pr_i(H|0,0)}{Pr_i(H|1,0)} < \frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \\ \text{Log probabilistically linear:} & \frac{Pr_i(H|1,1)}{Pr_i(H|1,0)} = \frac{Pr_i(H|0,1)}{Pr_i(H|0,0)} \text{ or } \frac{Pr_i(H|0,0)}{Pr_i(H|1,0)} = \frac{Pr_i(H|0,1)}{Pr_i(H|1,1)} \end{array} \quad (31)$$

Therefore, since cross difference (30) is also the determinant for log-probabilistic complementarity/-substitutability, sufficient conditions for convexity/concavity of the cost function can be re-expressed in terms of log-probabilistic complementarity/substitutability in Corollary 7.1.

Corollary 7.1. *With symmetric incentive efficiency for a partial effort (Assumption 6) and zero cross difference for the probability function (7), for agent i, the cost function*

- *is convex* ($\frac{c}{I_j(\boldsymbol{R}_j^*|1,1)} < \frac{c}{I_j(\boldsymbol{R}_j^*|1,0)}$) *if efforts are log probabilistic complements ((31) > 0).*
- *is concave* ($\frac{c}{I_j(\boldsymbol{R}_j^*|1,1)} > \frac{c}{I_j(\boldsymbol{R}_j^*|1,0)}$) *if efforts are log probabilistic substitutes ((31) < 0).*
- *is linear* ($\frac{c}{I_j(\boldsymbol{R}_j^*|1,1)} = \frac{c}{I_j(\boldsymbol{R}_j^*|1,0)}$) *if efforts are neither log probabilistic substitutes nor complements ((31) = 0).*

Corollary 7.1 shows log probabilistic complementarity (substitutability) being a sufficient condition for convexity (concavity) of the expected cost, and hence a part of the sufficient condition leads to the optimal effort pair.

Moreover, as inequality $\frac{c}{I_j(\mathbf{R}_j^*|1,1)} \lesseqgtr \frac{c}{I_j(\mathbf{R}_j^*|1,0)}$ compares the expected cost of inducing high effort from agent j if a high effort or a low effort is induced for agent i, it also helps us to understand the expected cost of inducing full instead of partial efforts, as shown in (20). Since there is always an extra cost of inducing a high effort from agent i rather than zero effort, no matter how much effort is induced from agent j, there exists a trade-off between the expected costs for these two agents. Namely, with log probabilistic complementarity, if the principal induces a partial effort from agent

[15] Similar to our interpretation for Assumption 7, we can also interpret it from the perspective of noisiness within signals if we switch around the order of $\log(Pr_i(H|1,0))$ and $\log(Pr_i(H|0,1))$.

[16] This is also referred to as log-supermodularity or log-submodularity (Athey, 2002).

i, on the one hand, the principal will save on the payment to agent i, but on the other hand, the principal will have to pay agent j more. On the contrary, if efforts are log probabilistic substitutes, the principal is more willing to induce a partial effort from agent j, as it means that the principal saves costs on paying both agent i and agent j. This suggests that the signal pair (0,1) is more informative for the principal when efforts are log probabilistically substitutable rather than log probabilistically complementary. Namely, log complementarity makes it more costly for the principal to deduce what the actual effort of agent j is in case there is interference from agent i's partial effort. On the contrary, for log-substitutability, the principal can use the fact that agent i is doing zero effort to infer the actual effort invested by agent j.

However, the above analysis focuses on the cost side; another component that affects the optimal effort choice is the revenue side, which complicates the analysis because the added revenue may also depend on spillover intensity/noisiness. These issues are addressed in the following subsection.

4.3.3 Change within added revenue and its link to the optimal effort pair

Similar to (27), (32) determines whether the expected revenue is concave, convex, or linear if it is smaller, larger, or equal to zero.

$$\begin{aligned} &\left\{\sum_{k=1}^{4} F_i(\mathbf{R}_{k,i},1,1)Pr_i(\mathbf{R}_{k,i}|1,1) - \sum_{k=1}^{4} F_i(\mathbf{R}_{k,i},0,1)Pr_i(\mathbf{R}_{k,i}|0,1)\right\} \gtreqless \\ &\left\{\sum_{k=1}^{4} F_i(\mathbf{R}_{k,i},0,1)Pr_i(\mathbf{R}_{k,i}|0,1) - \sum_{k=1}^{4} F_i(\mathbf{R}_{k,i},0,0)Pr_i(\mathbf{R}_{k,i}|0,0)\right\}, \end{aligned} \tag{32}$$

The left-hand side in (32) is the difference in revenue between inducing a high effort from agent j while a high effort or a zero effort from agent i, whereas the right-hand side is the difference in revenue between inducing a low effort from agent j while a high effort or a zero effort from agent i. We add the following simplifying assumption on the revenue function:

Assumption 8. *When effort 0 is induced from agent i and effort 1 from agent j, or the other way around, the expected revenue of the principal is the same each time:*

$$\sum_{k=1}^{4} F_i(\boldsymbol{R}_{k,i},1,0)Pr_i(\boldsymbol{R}_{k,i}|1,0) = \sum_{k=1}^{4} F_i(\boldsymbol{R}_{k,i},0,1)Pr_i(\boldsymbol{R}_{k,i}|0,1)$$

This means that the expected revenue from inducing (0,1) is the same as from inducing (1,0). This allows us to re-express the determinants for convexity/concavity (32) in the form of complementarity/substitutability, as shown in the following Definition 10.

Definition 10. *The revenue cross difference is defined as:*

$$\left\{\sum_{k=1}^{4} F_i(\boldsymbol{R}_{k,i},1,1)Pr_i(\boldsymbol{R}_{k,i}|1,1) - \sum_{k=1}^{4} F_i(\boldsymbol{R}_{k,i},1,0)Pr_i(\boldsymbol{R}_{k,i}|1,0)\right\} - \left\{\sum_{k=1}^{4} F_i(\boldsymbol{R}_{k,i},0,1)Pr_i(\boldsymbol{R}_{k,i}|0,1) - \sum_{k=1}^{4} F_i(\boldsymbol{R}_{k,i},0,0)Pr_i(\boldsymbol{R}_{k,i}|0,0)\right\} \gtreqless 0 \tag{33}$$

Similar to Definition 9, if the revenue cross difference equals zero, then the revenue function is linear. Otherwise, when (33) is positive (negative), we say that efforts are overall complementary (substitutable), as the revenue function contains both elements from the production function and from the probability function. Similar to Lemma 7, we establish Lemma 8 as a sufficient condition for optimal incentive schemes on the revenue side.

Lemma 8. *With symmetric revenue for a partial effort (Assumption 8), the revenue function*

- *is convex if efforts are overall substitutes ((33) < 0).*
- *is concave if efforts are overall complements ((33) > 0).*
- *is linear if efforts are neither overall substitutes nor complements ((33) = 0).*

Similar to subsection 4.2, we identify the detailed impact of spillover effects' intensity, noisiness and shape of production function on the optimal effort pair through their effect on revenue cross difference by dinstinguishing between two different scenarios for the revenue function, depending on whether revenue directly depends on efforts or on whether revenue stochastically affects signals that enter the revenue function. This is shown in detail in Section 1 of the supplementary materials. The case of effort pairs directly entering the production function is straightforward, as shown in (22), where the concavity (convexity) is purely determined by the shape of the production function, as we express in Case 1 in Section 1 of the Supplementary Materials. Therefore, under this scenario, spillover effects and noisiness only impact the optimal effort pair through the cost function as pointed out in Proposition 4a) below.

However, the case where signal pairs enter the production function is more complex as it is also determined by the shape of the probability function (23). Hence, we study two extreme cases to provide some insight into this scenario: either focusing on the impact of the probability function, assuming the production function has a constant margin, or focusing on the impact of the production function, assuming the probability function has a constant margin. The first setting is the same as 4.2.2, which delivers a more stringent condition for inducing full efforts, and therefore provides a sufficient condition for the results in Lemma 5. After taking some simplifying assumptions, results exhibited in Table 1 and Table 2 of the Supplementary Material reveal probabilistic cross difference, directions of spillover effect, and production cross difference as the determinants of revenue cross difference in different scenarios.

Specifically, when zero cross difference for the production function (Assumption 5) is assumed (Case 2(a) in Section 1 of the Supplementary Material), efforts are overall complements (substitutes) if both agents have positive (negative) spillover effects. For asymmetric spillover effects, the direction of the inequality in (33) is then determined by the average probabilistic cross difference.[17] When zero cross difference for the probability function (Assumption 7) is assumed (Case 2(b) in Section 1 of supplementary materials), complementarity/substitutability is determined by the productive cross difference: if both agents share symmetric spillover effects, then the revenue cross difference is opposite to the productive cross difference. Otherwise, if two agents have asymmetric spillover effects, then the revenue cross difference is opposite to the productive cross difference.[18]

In summary, despite different cases having various implications, they share the same determinants as they are jointly controlled by productive cross difference and spillover effect.

4.3.4 Sufficient conditions for optimal effort pairs and their link to spillover effects

After applying Lemma 7 and Lemma 8, we draw the link he link between convexity/concavity on the one hand, and probabilistic cross difference and revenue cross difference on the other hand, and hence, summarize the sufficient conditions under which inducing full efforts (a partial effort) is optimal as[19]:

Proposition 4. *Under Assumption 6 and 7, it is optimal to induce full efforts if*

a) efforts are not log probabilistic substitutes, and

b) efforts are not overall substitutes.

Proposition 4(a) describes the sufficient condition under which full efforts (a partial effort) are optimal from a cost perspective if the revenue function is convex or linear, and Proposition 4(b) ensures the sufficient condition under which full efforts (a partial effort) are optimal from a revenue perspective if the revenue function is concave or linear. Note that *a*) corresponds to the cost side condition stated in Lemma 6, which has a connection with the comparative statics results in Corollary 4.1. As (27) adds an extra component compared to (20), it poses more stringent conditions for inducing full efforts from the cost side, and therefore log probabilistic complementarity of the effort

[17] See Table 1 in from the Supplementary Material for further details.

[18] See Table 2 from the supplementary material for further details.

[19] Note again that the conditions in Proposition 4 are sufficient, but not necessary conditions, so that there may exist other scenarios where full efforts (a partial effort)are optimal.

is a sufficient condition for Corollary 4.1 after assuming zero cross difference for probability function (Assumption 7).

Thus, as we claimed in Subsections 4.3.2 and 4.3.3, Proposition 4a) and b) are in fact sufficient conditions for a convex (concave) cost function and a concave (convex) revenue function, which leads to the optimal effort pair. Proposition 4 in this manner also specifies how spillover effects and noise determine the optimal effort pair.

In general, this section highlights the determinants of revenue-cost trade-offs between inducing full efforts and a partial effort through convexity and concavity of the revenue and the cost function, which depend on the (absolute) intensity of spillover effects and the noise in signals. The analysis shows that firms are more willing to induce full efforts when added revenue is high and extra cost is low, namely, 1) if there are more intense spillover effects for full efforts and less intense spillover effects for a partial effort, and spillover effects are less intense for negative spillover effects in absolute terms; 2) if the signal is noisier for a partial effort and less noisy for full efforts. In general, full efforts is the optimal effort pair if efforts are not probabilistically substitutable and are not overall substitutable, which is decided jointly by the productive cross difference and spillover effects or noisiness.

5 Conclusion

This article develops a principal-agent framework with heterogeneous spillover effects, including the possibility of asymmetric spillover effects, enabling us to characterize optimal incentive schemes for firms with cross-department spillover effects. We demonstrate that the classic principal-agent setting is a special case within our framework when informational independence and no spillover effects are assumed, and that optimal contracts must rely on the most cost-efficient signal pair, which is determined by the type of spillover effects. For instance, with asymmetric spillover effects, for a firm with a technological and a compliance department, the technological department should only be rewarded if both departments report high signals (a collective performance evaluation scheme) as the technological department has a positive spillover effect on the compliance department, and the compliance department should only be rewarded if its signal is high and the other department's is low (a relative performance evaluation), as the technological department has a negative spillover effect on the compliance department.

Under our framework, the optimal effort pair is endogenous as the effort pair that maximizes the firm's expected profit, determined by the relative strength of spillovers, the noisiness of the signals, and revenue cross-difference. We derive a sufficient condition under which full efforts are optimal, namely that efforts are neither probabilistic substitutes nor overall substitutes. This sufficient condition can be interpreted in terms of spillover effects: if the spillover effect of one agent can be further intensified by the high effort from the other agent, then it is optimal to induce full efforts after taking the overall complement of efforts for revenue as given.

For instance, take the example of a firm with positive spillover effects between the technological and the marketing department. Assuming high effort by the technological department is indispensable, so that the principal only considers whether to induce full efforts or a partial effort from the marketing department, comparative statics concerning both costs and revenues show that full efforts become relatively more profitable if the two departments reinforce each other under full efforts. Intuitively, intensive marketing is especially valuable when the technology department has developed a high-quality product: a better product gives marketing more persuasive material to promote, while stronger marketing generates more users, feedback, and market visibility for the product and hence speeds up the technological advancement. On the other hand, full efforts from the marketing department are more attractive if inducing low effort from the marketing department makes it harder for the principal to conclude from a low signal of the technological department that this signal is due to low effort of the technological department; or if inducing high effort from the technological department makes it easier for the principal to infer from a high signal received about the marketing's department that the marketing department did high, and not low effort.

We believe the following extensions to be valuable topics for future research. First, the impact of heterogeneous spillover effects through informational spillovers, namely, when there is a correlation between the precision with which signals are received. Second, the effect of switching from discrete to continuous effort levels. Third, an extension of our model to a multiple-agent scenario ($n \geq 3$). Although the optimal payment for a multiple-agent scenario is illustrated in the Supplementary Material, identifying the optimal effort pair is complicated by multiple heterogeneous spillover effects, and we consider only an extreme case for simplicity. Hence, more effort is required to extend our model fully to a more general setting.

As for the real-world application of this paper, our model aligns with the setting of firms with multiple departments, where feedback loops create heterogeneous dependencies across departments. These examples suggest practical guidance for managers designing incentive schemes in environments with spillover effects.

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